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Series study of the one-dimensional 'true' self-avoiding walk

A L Stella†, S L A de Queiroz‡, P M Duxbury and R B Stinchcombe
Department of Theoretical Physics, 1 Keble Road, Oxford OX1 3NP, UK

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Abstract. The 'true' self-avoiding walk problem is formulated using a grand canonical approach, and exact enumeration methods are used to calculate the average end-to-end distance for one-dimensional 'true' self-avoiding walks with up to 21 steps. The results are in agreement with a universality picture obtained both from Monte Carlo simulations and from scaling and crossover arguments. The extrapolated value of the end-to-end distance exponent ν is $\nu = 0.67 \pm 0.04$.

1. Introduction

Among the problems currently under study in the context of lattice statistics, the 'true' self-avoiding walk (TSAW), recently introduced by Amit *et al* (1983), is one that exhibits some unusual features. The TSAW is defined as the problem of the traveller who steps at random but *tries to avoid* sites that have already been visited. It is quite different from the usual 'self-avoiding walk' ('self-repelling chain' (SRC) in the terminology of Amit *et al* (1983)), which is a random walk with *no self-intersections allowed*, well known as a model for the configurations of polymers in good solvents (see e.g. de Gennes 1979). Indeed TSAWs and SRCs differ in their critical exponents and even in their upper critical dimensionalities d_c ($d_c = 2$ from the TSAW while $d_c = 4$ for the SRC (de Gennes 1979)), in the absence or presence of configurations which trap the traveller and in the number and stability of fixed points displayed in their respective phase diagrams (de Queiroz *et al* 1984, see also below).

The problem is defined as follows (Amit *et al* 1983): on a lattice, the traveller has to move to one of the z nearest neighbours of the site he is at. The probability P_i of moving to a point i depends on the number of times n_i this site has already been visited:

$$P_i = \mathrm{e}^{-gn_i} \Big/ \sum_{j=1}^{z} \mathrm{e}^{-gn_j}. \tag{1}$$

The parameter g defines the strength with which the walk avoids itself ($g > 0$). Two remarks are worth making:

(i) At each step, all the past history of the walk enters into the determination of the probability distribution; this extremely complicated, non-Markovian, nature of the TSAW is actually the feature that makes it distinct from other problems, as we shall see below.

† Present and permanent address: Dipartimento di Fisica e Unità GNSM del CNR dell' Università di Padova, Padova, Italy.

‡ On leave from Departamento de Fisica, PUC, 22452 Rio de Janeiro, Brazil. Supported by CNPq.

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(ii) For $g = 0$ one recovers the ordinary random-walk (RW), for all space dimensionalities; on universality grounds we expect the asymptotic behaviour to be the same for every finite, non-zero g. For instance the average end-to-end distance for an N-step walk, $\langle R_N^2 \rangle^{1/2}$, must scale as N^ν, ν being the same for every $g > 0$ at a given dimension, and different from the random-walk value ($\nu = \frac{1}{2}$) for $d < d_c = 2$. As regards point (ii) above, Pietronero (1983) indeed obtained from a self-consistent approach to the TSAW both $d_c = 2$ and an explicit, approximate expression for ν, valid for every finite, non-zero g and $d \leqslant 2$, namely:

$$\nu = 2/(d+2) \qquad d \leqslant 2. \tag{2}$$

The above expression is a reminder that even in one dimension the TSAW is non-trivial: it gives in this case $\nu = \frac{2}{3}$, different from both the one-dimensional SRC exponent, $\nu = 1$ (McKenzie 1976) and the random-walk exponent ($\nu = \frac{1}{2}$). This result is in good agreement with Monte Carlo simulations, performed in 1D respectively for $g = 0.1$, 1.0 and 3.0 (Bernasconi and Pietronero 1983), where the authors find an overall picture consistent with $\nu = 0.67 \pm 0.01$ for all three values of the repulsion parameter.

This universality picture has been independently supported by the scaling and crossover arguments for the 1D TSAW of de Queiroz *et al* (1984). Therein, it is recalled that only in 1D is the $g \to \infty$ limit of the TSAW the SRC (in higher dimensionlities this equivalence does not hold, because 'self-trapped' configurations which are dead ends for SRCs have a way out in the TSAW case, as can be seen from normalisation condition (1) above); with $x \equiv e^{-g}$, the crossover between TSAW and SRC ($x \sim 0$) is given special attention. The random-walk limit ($x \to 1$) is discussed as well, the resulting renormalisation group flow diagram being consistent with the existence of an attractive fixed point located at some x_c between zero and one. This then implies that the exponent for the TSAW in 1D is the same for any finite, non-zero value of the repulsion parameter g.

In the present work we make use of series analysis both to discuss the numerical value of the exponent ν for the TSAW in one dimension and to check the consistency of the universality picture sketched above; this in turn enables us to comment on the specific features of series calculations for this problem, where crossover phenomena occur. We also comment on the relationship between the very peculiar way in which probability distributions are built for the TSAW and uncertainties in Monte Carlo and series results. In § 2 we discuss a grand-canonical formulation for the TSAW problem; in § 3 the results obtained from series are presented; in § 4 we discuss the influence of crossover and finite-size probability distribution on our results, comparing them with those obtained from Monte Carlo simulations; finally, concluding remarks are presented.

2. Grand-canonical formulation

In this section we get into details of the scheme outlined in de Queiroz *et al* (1984), pointing out its generalisations and limitations.

We denote a generic walk by α, N_α being its number of steps and $(R_\alpha^2)^{1/2}$ its end-to-end distance. Each walk will have an intrinsic probability, given by the product of the probabilities for each of its steps, and depending on $x \equiv e^{-g}$:

$$P(\alpha, x) = \prod_{i=1}^{N_\alpha} P_i \tag{3}$$

where the P_i are given by (1) above. The condition that at the Nth step the traveller has to be somewhere reflects itself in the normalisation:

$$\sum_{\alpha, N_\alpha = N} P(\alpha, x) = 1 \tag{4}$$

for any fixed N.

In order to formulate the problem in terms of a grand-canonical ensemble, we associate a fugacity K to each step of a walk (Shapiro 1978, de Gennes 1979). A walk α must have its weight W_α depending both on its intrinsic probability $P(\alpha, x)$ and on its number of steps, so we write:

$$W_\alpha = W(\alpha, x) K^{N_\alpha}. \tag{5}$$

Further, we *choose* to write $W(\alpha, x)$ as:

$$W(\alpha, x) = P(\alpha, x) / P(\alpha_{\max}, x) \tag{6}$$

where $P(\alpha_{\max}, x)$ is the intrinsic probability of the walk $\alpha_{\max}$ with the *same* number of steps as α and *maximal* end-to-end distance (for any lattice with inversion symmetry, this means $(R^2_{\alpha_{\max}})^{1/2} = N_\alpha$ in lattice parameter units). The choice is sensible because:

(i) It is certainly desirable that $W(\alpha, x)$ be proportional to $P(\alpha, x)$.

(ii) In the random-walk limit ($x = 1$), $W(\alpha, 1) = 1$ for every walk so the usual random-walk statistics is recovered.

(iii) In the limit $x \to 0$, it is only in one dimension that the SRC limit is recovered, again with the usual weights (weight one for a walk without self-intersections and zero for one with any number of self-intersections). For $d > 1$ the statistics of TSAWs with $x = 0$ is known to differ from that of SRCs (Pietronero 1983, de Queiroz *et al* 1984) and this fact reflects itself in that, with the above choice of weights:

(*a*) 'Self-trapped' configurations, which are dead ends for SRCs are not for TSAW. So, e.g., in figure 1(*a*) the TSAW can proceed with further steps from point A without the weight becoming zero as in the SRC case.

(*b*) Non-intersecting configurations where nearest-neighbour sites are visited not sequentially, which have weight one for SRCs, have a weight $\neq 1$ for TSAWs. For instance in figure 1(*b*) the step B → C has a probability $(2+2x)^{-1}$, because A has already been

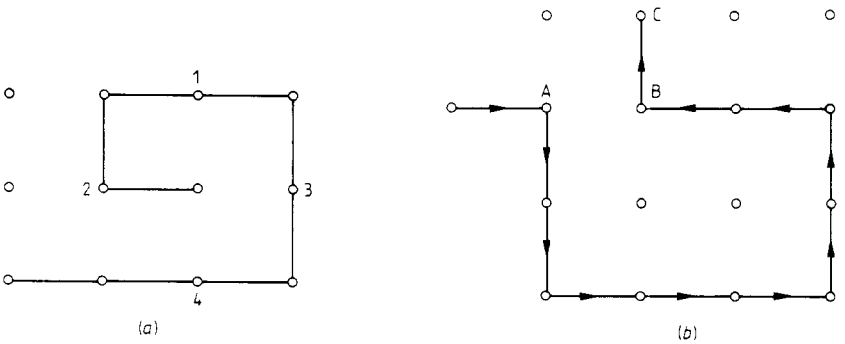

Figure 1. (*a*) This 'self-trapped' configuration is a dead end for SRCs, whereas for a TSAW the traveller can proceed to either 1, 2, 3, or 4 with equal probability. (*b*) This configuration has weight one for SRCs; for TSAWs the fact that A has been visited modifies the probability of the step B → C (see text), leading to a weight different from one.

visited; this is different from the probability $(3+x)^{-1}$ for any step but the first in α_{max} on a square lattice. Sensible as it is, the choice of weights given by equation (6) has some definite consequences as regards the 'susceptibility' exponent γ, defined in analogy with the SRC case (McKenzie 1976, de Gennes 1979) as related to the divergence of the grand canonical generating function:

$$G(x, K)=\sum_{N}\sum_{\alpha, N_\alpha=N} W(\alpha, x)K^{N_\alpha}\sim(K_c(x)-K)^{-\gamma}, \qquad K\to K_c(x)^- \tag{7}$$

where $K_c(x)$ is a lattice- and x-dependent quantity. The assumption of (7) implies

$$\sum_{\alpha, N_\alpha=N} W(\alpha, x)\sim K_c(x)^{-N}N^{\gamma-1} \qquad (N\to\infty). \tag{8}$$

On the other hand, it is easy to see from normalisation condition (4) and definition (6) that, if z is the number of nearest neighbours of a site:

$$\sum_{\alpha, N_\alpha=N} W(\alpha, x)=z(z-1+x)^{N-1} \tag{9}$$

where account has been taken of the fact that one has z equally probable choices for the first step of an α_{max} walk, and each subsequent step of the α_{max} path has probability $(z-1+x)^{-1}$. Equation (9) then implies that

$$\gamma=1; \qquad K_c(x)=(z-1+x)^{-1} \tag{10}$$

for any space dimensionality. These results deserve a few comments.

In polymer literature, the inverse of K_c is termed 'connective constant', its value giving an average of the number of alternatives available for each next step; the factor $N^{\gamma-1}$ in the expression analogous to equation (8) is the 'enhancement factor' (de Gennes 1979), reflecting the fact that for an SRC the condition of non-intersection 'pushes' the chain into regions of lower local density. A unitary value of γ and the corresponding absence of the enhancement factor would then be related to the applicability of a mean-field picture.

In the present case, although equation (10) gives $K_c(1)=z^{-1}$ which is the correct connective constant for a random walk, the interpretation of $K_c(x)$ for $x<1$ as a connective constant is less obvious, because different walks have different intrinsic weights, depending on all details of their past history even in the case $x=0$.

The unitary value of γ in all dimensions simply follows from the definition of weights (6) above. Whether it is a 'real' susceptibility exponent (in the same sense as the γ exponent of the generating function of SRCs is the susceptibility exponent of the zero-component classical vector model (de Gennes 1979)) is a matter that could be made clear if a correspondence could be drawn between the TSAW and a magnetic problem, in the spirit of the isomorphism between SRCs and the zero-component vector model. This 'magnetic' aspect of the problem must be more carefully studied. However it must be noticed that in one dimension the unitary value of γ can be understood on purely geometric grounds, while for $d\geq 2$ it is again correct that $\gamma=1$, this time because the critical dimensionality is two.

Throughout this work we shall make use of definition (6) above, its limitations as pointed out in the preceding paragraphs being of no importance here because we restrict ourselves to the 'thermal' aspects of the one-dimensional problem. As we try to make clear below, the essential ingredient that distinguishes the geometrical properties of the TSAW from those of other problems is the infinite-range, cumulative, memory

effect; provided that it is incorporated in a sensible way (which equation (6) certainly does), one can properly address questions such as the distribution of end-to-end lengths.

3. Series results

In this section we report the results of series calculations for the TSAW in one dimension, in which walks of up to 21 steps were taken into account; the weights used were those defined in equation (6). Several values of the repulsion parameter x were considered, namely $x = 0.1$, 0.3, 0.4, 0.5, 0.6, 0.7 and 0.9. We considered the attractive case ($x > 1$) as well, having found out that the walk is self-trapping for all $x > 1$ (as could be expected on intuitive grounds). We shall not comment further on the attractive case; instead we concentrate on what was found for $0 < x < 1$.

For each value of $N (1 \leq N \leq 21)$ and x (as quoted above) we calculated, through exact enumeration, the average both of the absolute value of the end-to-end distance $\langle |R_N(x)| \rangle$ and the square of the end-to-end distance $\langle R_N^2(x) \rangle$. These are given by

$$\langle |R_N(x)| \rangle = \sum_{\alpha, N_\alpha = N} W(\alpha, x) |R_\alpha| \Big/ \sum_{\alpha, N_\alpha = N} W(\alpha, x) \tag{11}$$

and analogously for $\langle R_N^2(x) \rangle$. From (9) above one has $\Sigma_{\alpha, N_\alpha = N} W(\alpha, x) = 2(1+x)^{N-1}$ in the one-dimensional case; in table 1 we display the values of $\langle |R_N(x)| \rangle$ for $x = 0.1$, 0.3, 0.5, 0.7 and 0.9; table 2 shows $\langle R_N^2(x) \rangle$ for the same values of x.

We have analysed our data in two different ways: one is the classical ratio method; the second is an analysis of moments based on the assumption of a particular scaling

Table 1. The values of $\langle |R_N(x)| \rangle$ for $N \leq 21$ and $x = 0.1$, 0.3, 0.5, 0.7 and 0.9.

N \ x	0.1	0.3	0.5	0.7	0.9
1	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000
2	1.818 182	1.538 462	1.333 333	1.176 471	1.052 632
3	2.652 893	2.183 432	1.888 889	1.692 042	1.554 017
4	3.418 185	2.694 709	2.237 037	1.894 044	1.618 320
5	4.190 593	3.250 192	2.692 840	2.295 571	1.995 975
6	4.910 256	3.726 975	3.025 844	2.499 166	2.064 905
7	5.634 967	4.230 464	3.429 849	2.844 969	2.381 129
8	6.313 542	4.676 756	3.747 255	3.044 860	2.452 164
9	6.996 168	5.144 072	4.119 441	3.356 477	2.730 250
10	7.638 025	5.564 540	4.422 575	3.551 463	2.802 229
11	8.283 594	6.004 407	4.772 131	3.839 577	3.053 788
12	8.892 789	6.403 851	5.063 101	4.029 595	3.126 107
13	9.506 408	6.821 733	5.394 938	4.300 228	3.357 893
14	10.087 304	7.204 081	5.675 538	4.485 587	3.430 216
15	10.672 814	7.603 980	5.992 979	4.742 471	3.646 554
16	11.228 574	7.971 622	6.264 534	4.923 556	3.718 688
17	11.789 385	8.356 256	6.569 931	5.169 212	3.922 536
18	12.323 002	8.711 259	6.833 513	5.346 387	3.994 367
19	12.861 938	9.082 596	7.128 567	5.582 624	4.187 853
20	13.375 927	9.426 504	7.385 055	5.756 214	4.259 315
21	13.895 414	9.786 164	7.671 063	5.984 387	4.444 029

Table 2. The values of $\langle R_N^2(x)\rangle$ for $N \leq 21$ and $x = 0.1$, 0.3, 0.5, 0.7 and 0.9.

N \ x	0.1	0.3	0.5	0.7	0.9
1	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000
2	3.636 364	3.076 923	2.666 667	2.352 941	2.105 263
3	7.611 570	5.733 728	4.555 556	3.768 166	3.216 066
4	12.846 888	9.030 746	6.844 444	5.416 420	4.402 990
5	19.226 479	12.801 790	9.351 605	7.140 128	5.597 767
6	26.714 104	17.129 893	12.188 313	9.046 576	6.854 083
7	35.192 281	21.824 615	15.189 740	11.017 903	8.118 554
8	44.630 581	26.981 943	18.479 332	13.148 142	9.435 945
9	54.930 787	32.453 310	21.912 758	15.333 837	10.761 142
10	66.071 453	38.318 849	25.593 648	17.660 653	12.133 436
11	77.965 485	44.460 571	29.407 725	20.038 351	13.513 115
12	90.602 119	50.949 896	33.440 568	22.542 267	14.935 554
13	103.902 489	57.684 608	37.595 379	25.094 403	16.365 038
14	117.864 016	64.732 573	41.948 610	27.761 000	17.833 857
15	132.415 302	72.004 241	46.414 614	30.473 446	19.309 465
16	147.559 026	79.558 952	51.062 073	33.291 177	20.821 610
17	163.230 916	87.319 986	55.815 318	36.152 540	22.340 349
18	179.438 315	95.341 208	60.735 837	39.111 735	23.893 286
19	196.121 406	103.552 478	65.756 034	42.112 627	25.452 659
20	213.292 070	112.005 368	70.931 896	45.205 057	27.044 244
21	230.894 628	120.634 937	76.201 870	48.337 531	28.642 127

form for the end-to-end length distribution, proposed by McKenzie and Moore (1971) for SRCS.

Although the results obtained through both procedures show a certain amount of spread, they give strong support to the universality picture of Bernasconi and Pietronero (1983) and of de Queiroz *et al* (1984).

3.1. Classical ratio method

Assuming, for large N

$$\langle |R_N(x)|\rangle = A_1(x)N^{\nu}, \qquad \langle R_N^2(x)\rangle = A_2(x)N^{2\nu} \tag{12}$$

where we expect ν to be the same for every $0 < x < 1$, one has

$$\frac{\langle |R_{N+2}(x)|\rangle}{\langle |R_N(x)|\rangle} = \left(1+\frac{2}{N}\right)^{\nu}, \qquad \frac{\langle R_{N+2}^2(x)\rangle}{\langle R_N^2(x)\rangle} = \left(1+\frac{2}{N}\right)^{2\nu} \tag{13}$$

where the ratios are taken between terms of the same parity in order to eliminate the characteristic even–odd fluctuations that occur in loose-packed lattices (Watts 1974). From (13) one obtains the following estimates, which will depend on N and x (the x-dependence being implicitly understood from now on):

$$\nu_N = \ln(\langle |R_{N+2}|\rangle/\langle |R_N|\rangle)/\ln(1+2/N) \tag{14a}$$

$$\nu_N = \tfrac{1}{2}\ln(\langle R_{N+2}^2\rangle/\langle R_N^2\rangle)/\ln(1+2/N). \tag{14b}$$

The values of ν_N found from $\langle R^2\rangle$ series using (14*b*) are displayed against $1/N$ in figure 2.

A qualitatively similar behaviour was found for ν_N from $\langle|R|\rangle$ series. The curvature of the $\nu \times 1/N$ plots increase as one approaches either the $x \to 0$ or the $x \to 1$ limit; the straightest plots are for $x = 0.5$ to 0.7. This is consistent with the existence of two crossovers (RW → TSAW at $x = 1$ and SRC → TSAW at $x = 0$) and with the location of the inferred fixed point on the phase diagram of de Queiroz *et al* (1984) at x_c around ~0.5–0.7. The generally high degree of curvature evident from figure 2 requires that care be taken in forming infinite-N extrapolants. Neville tables (see e.g. Gaunt and Guttman 1976) formed from the data indeed show considerable scatter. This is reflected in the errors in table 3, where we display the estimates of ν obtained using Neville tables, for several values of x and for $\langle R^2\rangle$ and $\langle|R|\rangle$ series. The results in table 3 are consistent with the 1D TSAW being in the same universality class for any x in the range $0 < x < 1$; our overall estimate for ν is

$$\nu = 0.69 \pm 0.03 \tag{15}$$

where the error bar reflects the scatter of the central estimates in table 3.

Table 3. The values of ν as from the ratio method; obtained, for each x, from Neville tables.

x	$\nu(R^2)$	$\nu(\|R\|)$
0.1	0.64 ± 0.05	0.66 ± 0.04
0.3	0.68 ± 0.04	0.68 ± 0.04
0.4	0.69 ± 0.04	0.68 ± 0.03
0.5	0.70 ± 0.03	0.68 ± 0.03
0.6	0.71 ± 0.02	0.70 ± 0.02
0.7	0.70 ± 0.03	0.70 ± 0.03
0.9	0.65 ± 0.04	0.66 ± 0.03

As a check on the above analysis, and in order to discuss the scaling properties of the TSAW, in the range $0 < x < 1$, we have performed the alternative analysis described below.

3.2. Analysis of moments of the distribution

In a paper on polymer statistics, McKenzie (1973) points out that, if the end-to-end probability distribution of a self-repelling chain has the asymptotic form (McKenzie and Moore 1971)

$$P_N(\boldsymbol{r}) \sim (r/R_N)^g \exp\{-(r/R_N)^\delta\}, \qquad r \gg R_N \tag{16}$$

where $R_N \sim N^\nu$ is a scaling length and $P_N(\boldsymbol{r})$ is the probability that an N-step walk start at the origin and end at $\boldsymbol{r}$, then $\delta = (1-\nu)^{-1}$ and $g = [d(\nu - \frac{1}{2}) + 1 - \gamma]/(1-\nu)$ (McKenzie and Moore 1971), where d, ν and γ are respectively space dimensionality, 'correlation length' and 'susceptibility' exponents. In this case the 'reduced moment' $\sigma_t(N)$ defined by

$$\sigma_t(N) \equiv \langle R_N^t\rangle / \langle R_N^2\rangle^{t/2} \tag{17}$$

must approach a well defined value σ_t as $N \to \infty$, as a consequence of the existence

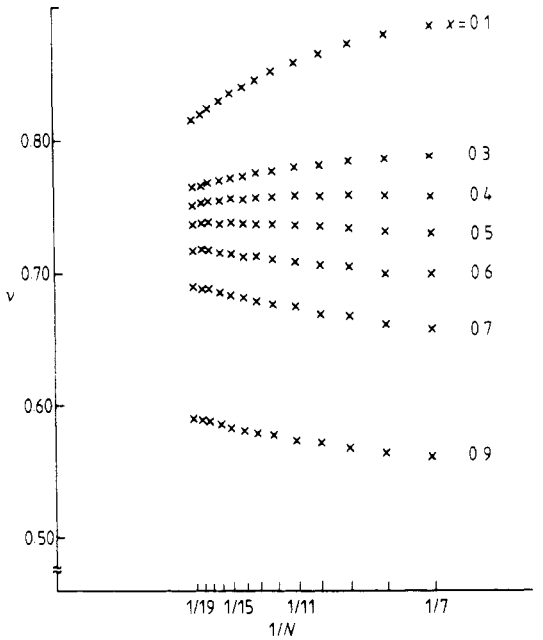

Figure 2. Plot of ν_N against $1/N$ for R^2 series.

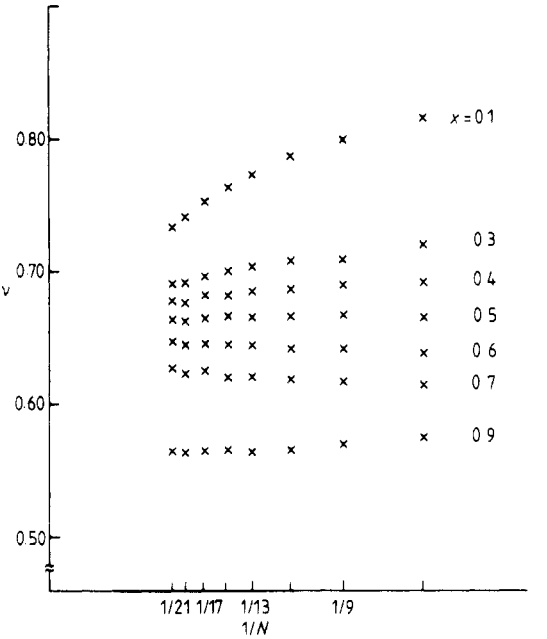

Figure 3. Plot of ν_n against $1/N$ as from the moment analysis. Only odd terms are used in order to avoid even–odd oscillations.

of a single scaling length; from (16) this value can be deduced to be:

$$\sigma_t = \frac{\Gamma((d+g+t)/\delta)}{\Gamma((d+g)/\delta)}\left(\frac{\Gamma((d+g)/\delta)}{\Gamma((d+g+2)/\delta)}\right)^{t/2} \tag{18}$$

(McKenzie 1973).

If we assume that a probability distribution of the form (16) holds in our case as well, and recall that $\gamma = 1$ in $d = 1$, we can use our data on $\langle |R_N| \rangle$ and $\langle R_N^2 \rangle$ to obtain estimates of

$$\sigma_1^2 = [\Gamma(\tfrac{3}{2} - \nu)]^2/[\Gamma(\tfrac{1}{2})\Gamma(\tfrac{5}{2} - 2\nu)] \tag{19}$$

and from these infer the value of ν.

The values of ν_N found from (17) and (18), for odd N and several values of x, are plotted against $1/N$ on figure 3. The overall behaviour is similar to that found using ratio analysis (see figure 2). However, there is a systematic shift to smaller values of ν and the spread of the ν_N estimates on varying x is somewhat smaller. Neville tables are again used to extrapolate the data. The results are displayed in table 4. As in the ratio analysis, there is some scatter in the Neville tables, as well as variation with x, especially for x near 0 and 1. From these results we estimate ν in the range $0 < x < 1$ to be

$$\nu = 0.65 \pm 0.03 \tag{20}$$

where the error bar reflects the scatter of the central estimates in table 4. This result is lower than, but still consistent with (15). This in turn enables us to state that a scaling form as (16) above provides a reasonable representation of the asymptotic behaviour of the 1D TSAW, at least for $0 < x < 1$.

Table 4. The values of ν as from the moment analysis; obtained, for each x, from Neville tables.

x	ν
0.1	0.62 ± 0.04
0.3	0.64 ± 0.03
0.4	0.65 ± 0.03
0.5	0.65 ± 0.03
0.6	0.65 ± 0.02
0.7	0.65 ± 0.02
0.9	0.60 ± 0.03

4. Discussion

The results for ν found from our series expansions are consistent both with the Monte Carlo results of Bernasconi and Pietronero (1983) and with the value of $\frac{2}{3}$ found by Pietronero (1983). Indeed the moment analysis of § 3.2, suggests that: (i) a scaling form $P_N(r) \sim (r/R_N)^g \times \exp\{-(r/R_N)^\delta\}$ holds asymptotically, which is one of Pietronero's assumptions; and (ii) the value $\nu = \frac{2}{3}$, which yields $g = \frac{1}{2}$ and $\delta = 3$, is consistent with our results.

Despite this agreement the uncertainties encountered in series extrapolations for the TSAW are larger than those encountered in studies of SRCS. For instance the 2D exact enumeration data of Grassberger (1982) shows that at $N = 21$ one is already within $\sim 0.5\%$ of the final extrapolated results, and that the curvature in the ν_N data is rather small. Typical estimated deviations of about 0.5% are found also in shorter series calculations of exponents for SRCS both in two and three dimensions (Martin and Watts 1971, McKenzie 1973, Watts 1974). Although not directly related to our problem, the 'spiral self-avoiding walk' of Privman (1983) shows a greater irregularity in the behaviour of the series, with a spread of $\sim 10\%$ in ν and 25% in γ from a 40-term series calculation.

Concerning Monte Carlo studies of SRCS, a fractional spread of about 0.5% is usually obtained from 10 000-step walks (in two dimensions, Havlin and Ben-Avraham (1983) quote $\nu = 0.753 \pm 0.004$ with this number of steps). On the other hand, Bernasconi and Pietronero (1983) felt it was necessary to go to 200 000 Monte Carlo steps in the 1D TSAW in order to make sure they could claim a fractional deviation of about 1% in their result.

The slow convergence of the series and Monte Carlo calculations for the 1D TSAW, we suggest, is due to two effects:

(i) The complicated, cumulative memory effects that appear in the walk weights (equation (1)). These effects make the attainment of a reasonably stable probability distribution more difficult than for example in the problem of self-repelling chains.

(ii) The presence of two crossovers in the problem. In order to check how these effects are smoothed out with increasing number of terms, it would be useful to analyse extended TSAW series, in one dimension (where both effects occur) as well as in dimensionalities greater than one (where this double crossover will not be present, but effect (i) will be). Preliminary results obtained in one dimension by Dekeyser (1984) confirm the overall picture obtained above.

In conclusion, our series study of the ID TSAW confirms the universality picture of Bernasconi and Pietronero (1983) and of de Queiroz *et al* (1984). We estimate ν to be $\nu = 0.67 \pm 0.04$ and suggest that the intermediate fixed point is located at $x_c = 0.6 \pm 0.1$.

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